AQUEOUS FLOW LIMITATION IN UNIFORM COLLAPSIBLE TUBES:
MULTIPLE FLOW-LIMITED FLOW-RATES AT THE SAME PRESSURE DROP
AND UPSTREAM TRANSMURAL PRESSURE

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INTRODUCTION
Large-amplitude oscillation in collapsible tubes is often but not necessarily associated with flow-rate limitation\(^1\). Flow limitation can arise through either of two mechanisms, one based on viscous head loss and the other on wavespeed\(^2\). The transition between the two appears to be seamless, so that flow limitation can occur at any flow-rate. In contrast, the self-excited oscillations which are often associated with flow limitation require fluid inertia for their maintenance, and are accordingly seen only above a Reynolds number based on uncollapsed tube diameter of a few hundred. The question of the minimum Reynolds number is of importance in numerical modelling, where stability or validity may be lost as the Reynolds number increases. Clinically, undesirable oscillations can also occur in the inferior vena cava if heart-lung bypass pumping is turned up too high\(^3\). Until now, the lowest Reynolds number for published oscillations was about 300, for an extremely diaphanous tube\(^5\). We have re-investigated this question, using ordinary latex Penrose tubes.

METHODS
A latex rubber tube of nominal inside diameter 6.35mm and measured wall thickness 0.268mm was mounted between rigid pipes 281.5mm apart. Since, unlike the rather thick-walled tubes on which this laboratory has previously concentrated, the tube in air was unable to support aqueous contents without severe sagging, it was externally submerged in water derived from the upstream circuit. The tube conveyed aqueous flow (rate \(Q\), measured by timed collection) propelled by upstream head \(p_u\) from an always overflowing reservoir. Pressures entering (\(p_1\)), leaving (\(p_2\)) and surrounding (\(p_e\)) the tube were measured by means of disposable blood-pressure transducers. For each of the flow-limitation curves, a different value of micrometer-controlled resistance (M2) just upstream of the tube was set. Points on a given curve were obtained by incremental increase of a second micrometer-controlled resistance (M1) just downstream of the flow-driving reservoir; thus each curve was traversed in the direction of decreasing \(p_{12} = p_1 - p_2\). This procedure produces curves at a not-quite-constant \(p_{e1} = p_e - p_1\); \(p_{e1}\) decreased systematically but slowly along each curve (by a maximum of less than 0.3 kPa for all but the four lowest-\(Q\) curves).

RESULTS
Figure 1 shows the resulting flow-limitation curves; for each curve, the Reynolds number of the oscillatory flow just prior to the onset of steady flow is indicated.

Figure 1 Pressure drop vs. volume flow-rate. The legend indicates resistance just upstream of the tube; a lower micrometer reading corresponds to a higher resistance.
The conversion of oscillatory to steady flow was accompanied at high flow-rates by a small increase in flow-rate. For instance the oscillatory flow at Re = 546 became a steady flow at Re = 624. Conversely, at low flow-rates, establishment of steady flow was accompanied by flow-rate reduction, e.g. from Re = 142 to Re = 122. Oscillation did not occur at Re = 50.

Figure 2 shows the corresponding values of the transmural pressure at the upstream end of the tube.

![Figure 2](image)

Figure 2 Upstream transmural pressure vs. flow-rate. All data points correspond to those shown in Figure 1.

**DISCUSSION**

Flow-rate is not the only criterion controlling the onset of self-excited oscillation in collapsible tubes. The stiffness of the tube wall plays a major role; thus in the thick-walled (2.4mm) silicone rubber tubes that we have characterised previously, oscillations start at a Reynolds number that is already well into the turbulent flow regime. It is also well known that the impedance presented by the apparatus downstream of the collapsible tube discourages the appearance of oscillations. This has generally been taken to mean the downstream resistance, but in fact the important quantity is the impedance at the point where viscous flow limitation gives way to viscous damping forces is exceeded. Nor can it be assumed that oscillations occur at the point where viscous flow limitation gives way to wavespeed flow limitation, because it has previously been shown that flow limitation can occur in the absence of oscillation, at flow-rates clearly indicating wavespeed limitation.

The procedure identifies a flow-rate below which oscillations do not occur for the given combination of tube and downstream impedance. Despite the fact that this tube has a wall which is stiffer in bending than that of the tube used by Ohba et al., the oscillations are maintained to a Reynolds number lower than in their experiments (which were not concerned specifically with the minimum flow-rate for oscillations). However, the minimum flow-rate at which oscillations occurred here proved highly sensitive to details of tube mounting, and in a subsequent experiment oscillations were sustained only down to Re = 260.

Above flow-limited flow-rate, the curves in Figure 1 appear to share the same relation between Q and p12(Figure 2). These results show that p12, even if it is in fact still in command, effectively fails experimentally as the controller of Qlim. To the resolution of the measurements made here, the several values of Qlim above 1.5 ml/s all pertain to the one value of p12, of about 0.1–0.2 kPa, when compared at a common p12. Only at the lower values of Qlim where p12 varies more rapidly with Qlim does a monotonic dependence become clear.

Thus multiple values of Qlim occur here at a single combination of p12 and p12, at least to the resolution of these measurements. It was previously thought that this behaviour was confined to tapered-stiffness tubes. It remains to be decided, perhaps through modelling, whether the dependence of Qlim on p12 at a given p12 in fact remains monotonic and determinate. However, for experimental purposes, it appears that the relationship becomes sufficiently close to indeterminate as to be problematic, giving at least the impression of flow-limited flow-rate non-uniqueness.