Fluid-structure interactions in the human upper airway — large-displacement biomechanics

Novak S. J. Elliott¹*, Anthony D. Lucey¹ and Matthias Heil²

¹ Fluid Dynamics Research Group, Curtin University of Technology, Perth, Australia
² School of Mathematics, University of Manchester, Manchester, U.K.

* Corresponding author. Email: n.s.j.elliott@curtin.edu.au

Abstract: Obstructive breathing disorders, such as sleep apnoea and snoring, interfere with normal respiration and sleep, reducing brain-oxygen saturation and are linked with hypertension and heart failure. The mechanics of the human upper airway are characterised by the interaction of the low-stiffness structures, being the soft palate and throat sidewalls, with the air flow producing soft-tissue vibrations that may lead to airway closure. In order to investigate this dynamical system we employ a cantilevered flexible plate (soft-palate) immersed in a two-dimensional channel (pharynx) flow. For this canonical analogue, we take the next step towards biomechanical realism by modelling finite-amplitude motions of the flexible plate and incorporating finite thickness in its structure. The structural model makes use of a geometrically nonlinear formulation of the solid mechanics. Viscous flow is modelled at Reynolds numbers giving unsteady laminar flow. The fully-coupled fluid-structure interaction (FSI) model is developed using the open-source finite-element library oomph-lib. We begin with a validation study of the structural mechanics through examining the effects of finite amplitude and finite thickness on the in-vacuo modes. Thereafter, we use the FSI model to illustrate both stable and unstable motions of the plate. This paper demonstrates the versatility of the new modelling approach and its suitability for characterising the dependence of the plate’s stability on the system parameters.

Keywords: Upper airway, plate mechanics, fluid-structure interaction

1 Introduction

A cantilevered flexible plate immersed in a two-dimensional channel flow has previously been shown to capture fluid-structure interactions (FSIs) representative of respiratory flow and soft-palate motion in the upper airway [1, 3, 6–8]; such an analogue FSI system is represented by Figure 1. These models generally utilize ideal flow giving very high Reynolds numbers but with viscous boundary-layer effects modelled implicitly through the imposition of a Kutta condition [3, 6, 7] or an applied channel resistance [1]. Recently investigators have modelled the effects of fluid viscosity explicitly for the laminar regime of Reynolds numbers [2] and implemented constant pressure-drop boundary conditions [11]. In all of these studies—spanning low to high Reynolds numbers—short plates (with low mass ratio) are shown to lose their stability to a single-mode flutter instability at a critical value of flow speed or Reynolds number based on channel height. The destabilization mechanism is fundamentally similar being due to an irreversible energy transfer from fluid to plate. This arises from a phase difference between fluid pressure and plate motion that owes its origin to the finite length of the flexible plate [2, 6, 10].

In all the aforementioned studies, linear structural mechanics were adopted by using the one-dimensional (1-d) Euler-Bernoulli beam equation. A nonlinear structural model has been developed by including an inextensibility condition but this utilized potential flow [9]. Plainly the soft palate undergoes displacements beyond the linear range, particularly during obstructive breathing disorders. It is also of non-negligible thickness and subjected to fluid friction in a viscous flow field. Accordingly, the present work includes these effects thereby yielding a more faithful biomechanical FSI model. To achieve this, we employ a continuum mechanics approach, developing a model of the cantilever as a two-dimensional (2-d) elastic solid beam immersed in viscous flowing fluid. As an intermediate step we also implement a (geometrically) nonlinear 1-d beam model. This paper describes the development of the improved FSI model then demonstrates its utility in simulating both stable and unstable motions of the flexible plate and the characterization of the system’s behaviour.

Australasian Congress on Applied Mechanics, ACAM 6
12–15 December 2010, Perth, Australia
2 Theoretical model

The 2-d flexible plate in viscous channel flow is shown schematically in Figure 1(a). All quantities with superscripted asterisks are dimensional and those without are nondimensional. The flexible plate is modelled as a 2-d elastic solid beam of length \( l^e \), thickness \( h^e \), density \( \rho^s \), elastic modulus \( E^e \) and Poisson's ratio \( \nu \). It is attached to a rigid wall of the same thickness and length \( l^r \) that is positioned at midspan of the surrounding channel, which has length \( L^e \) and height \( H^e \). A fluid of density \( \rho^f \) and dynamic viscosity \( \mu^e \) is prescribed. Steady Poiseuille flow enters the domain through the upper and lower inlets, each having mean velocity \( U^e \); outflow is assumed to be parallel and axially traction-free.

For the analogous model with a 1-d beam the central rigid wall and beam coincide with the centreline of their 2-d counterparts.

3 Beam in vacuo: structural mechanics

3.1 Method

3.1.1 Governing equations

In the 1-d beam model we nondimensionalise all lengths and spatial coordinates on the beam length, \( L^e = l^e \), and all stresses and tractions on the effective 1-d elastic modulus, \( S^e = E^e / (1 - \nu^2) \). Beam deformation due to applied traction \( \mathbf{T} \) is governed by the principle of virtual displacements (PVD),

\[
\int_0^{l^e} \left[ \gamma \delta \gamma + \frac{h^2}{12} \delta \kappa - \left( \frac{1}{h} \sqrt{A/a} \mathbf{T} - A^2 \frac{\partial^2 \mathbf{R}}{\partial \xi^2} \right) \cdot \delta \mathbf{R} \right] ds = 0,
\]

where\( a = \frac{\partial \mathbf{r}}{\partial \xi} \cdot \frac{\partial \mathbf{r}}{\partial \xi} \) and \( A = \frac{\partial \mathbf{R}}{\partial \xi} \cdot \frac{\partial \mathbf{R}}{\partial \xi} \).

The Lagrangian coordinate \( \xi = \xi^e / l^e \) parametrises the beam’s undeformed shape [see Figure 1(b)] and \( \mathbf{r}(\xi) \) and \( \mathbf{R}(\xi) \) are the position vectors to a material particle on the beam’s centreline in the undeformed and deformed configurations, respectively. The terms \( a \) and \( A \) represent the squares of the lengths of infinitesimal material line elements in the respective undeformed and deformed configurations; i.e., ‘1 × 1 metric tensors’ of the beam’s centreline. The quantity \( \sqrt{A/a} \) represents the ‘extension ratio’ of the beam’s centreline, and \( ds = \sqrt{a} d\xi \). The \((1 \times 1)\) strain and bending ‘tensors’ \( \gamma \) and \( \kappa \) are given by

\[
\gamma = \frac{1}{2} (A - a) \quad \text{and} \quad \kappa = -(B - b),
\]

with \( b \) and \( B \) being the curvature of the beam’s centreline before and after deformation,

\[
b = \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial \xi^2}, \quad B = \mathbf{N} \cdot \frac{\partial^2 \mathbf{R}}{\partial \xi^2}.
\]
respectively, and \( \hat{n} \) and \( \hat{N} \) the corresponding unit normals. Thus (1) is a 1-d beam equation that takes into account curvature hence finite length, and axial stretching. Finally, 
\[
\Lambda = \frac{l_{\text{flexible}}}{T^*} \sqrt{\frac{\rho_s^*}{E_{\text{eff}}^*}} \tag{5}
\]
is the ratio of the natural timescale of extensional oscillations in the (solid) beam, \( T^*_s = l_{\text{flexible}} \sqrt{\rho_s^*/E_{\text{eff}}^*} \), to the timescale \( T^* \) used in the nondimensionalisation of the equations. Being in vacuo we simply choose \( T^* = T^*_s \), giving \( \Lambda = 1 \). The parameter \( \Lambda^2 \) may also be interpreted as the nondimensional beam density, thus by setting \( \Lambda = 0 \) we may ignore beam inertia.

For the 2-d beam model a pair of Lagrangian coordinates [see Figure 1(c)] parametrise the Eulerian position vector, \( R(\xi^1, \xi^2, t) \), and the PVD becomes
\[
\int \left\{ \sigma^{ij} \delta \gamma_{ij} + \Lambda^2 \frac{\partial^2 R}{\partial t^2} \cdot \delta R \right\} dv - \oint_{A_{\text{tract}}} T \cdot \delta R \, dA = 0, \tag{6}
\]
where \( \sigma^{ij} \) is the (symmetric) second Piola-Kirchhoff stress tensor, \( \gamma_{ij} \) is the Green strain tensor, \( dv \) is the differential unit of volume in the undeformed configuration, and \( A_{\text{tract}} \) is the deformed surface area over which acts the traction \( T \). We choose the same characteristic length as for the 1-d beam model, but the stresses and tractions are nondimensionalised on the ‘real’ (i.e. 3-d) elastic modulus, \( S^* = E^* \), thus
\[
\Lambda = \frac{l_{\text{flexible}}}{T^*} \sqrt{\frac{\rho_s^*}{E^*}}. \tag{7}
\]
As before the equations are nondimensionalised on the natural timescale of the solid so \( \Lambda^2 = 1 \).

### 3.1.2 Constitutive law

For ‘slender’ beam geometries large bending deflections do not create large strains, so a linearised relation between stresses and strains is appropriate. In our constitutive equation we form the tensor of elastic coefficients with the deformed metric tensor rather than the undeformed one. This yields a geometrically nonlinear formulation [e.g., (3a,b) in the 1-d beam model] as the strain depends nonlinearly on the displacements, capturing the kinematics of the deformation exactly for arbitrarily large displacements and rotations, though for small strains the difference between this and a linear theory is negligible. As the thickness of the beam increases relative to its length the strains on the top and bottom surfaces become significant at large bending deflections. Which constitutive equation one ought to use in the large strain regime is not straightforward and depends on the specific material being modelled. The work presented here focuses on beams slender enough for this not to be an issue, although the capability exists to use a full nonlinear constitutive relation (e.g., Mooney-Rivlin, Neo-Hookean).

### 3.1.3 Boundary and initial conditions

The 1-d and 2-d beams are clamped at the left (upstream) end and free at the right (downstream) end. For a linear Euler-Bernoulli cantilevered beam of length \( l \) and flexural rigidity \( B \) the \( n^{th} \) eigenmode of amplitude \( \eta_n \) is described by a vertical displacement profile \( \eta_n(x) \), which may be produced by applying the following distributed load
\[
B \nabla^4 \eta_n(x) = B \frac{\beta_n}{l} \left( \frac{\beta_n}{l} \right)^4 \left[ \cosh(\beta_n x/l) - \cos(\beta_n x/l) - \frac{S_n}{T_n} (\sinh(\beta_n x/l) - \sin(\beta_n x/l)) \right]; \tag{8}
\]
\( B = Eh^2/[12(1-\nu^2)] \) is the flexural rigidity of the beam, \( S_n = \cosh(\beta_n) + \cos(\beta_n), T_n = \sinh(\beta_n) + \sin(\beta_n) \) and \( \beta_n \) satisfy \( \cos(\beta_n) \cosh(\beta_n) = -1 \) (e.g., \( \beta_2 = 4.6941 \)). In the present work we apply \( T = (0, B \nabla^4 \eta_2(\xi)) \) to the 1-d beam and \( T = (0, B \nabla^4 \eta_2(\xi_{\text{top}})) \) to the 2-d beam as initial conditions in the steady problem.
Table 1: Dimensional parameter values.

<table>
<thead>
<tr>
<th>Type</th>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>L⁺</td>
<td>40.5 mm</td>
<td>Length of channel</td>
</tr>
<tr>
<td></td>
<td>H⁺</td>
<td>5.0 mm</td>
<td>Height of channel</td>
</tr>
<tr>
<td></td>
<td>l⁺_{rigid}</td>
<td>6.0 mm</td>
<td>Length of rigid central wall</td>
</tr>
<tr>
<td></td>
<td>l⁺_{flexible}</td>
<td>8.0 mm</td>
<td>Length of flexible plate</td>
</tr>
<tr>
<td></td>
<td>h⁺</td>
<td>0.25 mm</td>
<td>Thickness of flexible plate</td>
</tr>
<tr>
<td>Solid</td>
<td>ν</td>
<td>0.3333</td>
<td>Poisson's ratio of solid</td>
</tr>
<tr>
<td></td>
<td>ρ⁺_s/ρ⁺_f</td>
<td>84.2</td>
<td>Solid-to-fluid density ratio</td>
</tr>
<tr>
<td></td>
<td>B⁺</td>
<td>1.87 × 10⁻⁸ N·m</td>
<td>Flexural rigidity of flexible plate</td>
</tr>
<tr>
<td>Fluid</td>
<td>U⁺</td>
<td>0.08488 m/s</td>
<td>Mean inlet velocity</td>
</tr>
<tr>
<td></td>
<td>ρ⁺_f</td>
<td>1.1774 kg/m³</td>
<td>Density of fluid</td>
</tr>
<tr>
<td></td>
<td>µ⁺</td>
<td>1.98 × 10⁻⁵ kg/(m·s)</td>
<td>Dynamic viscosity of fluid</td>
</tr>
</tbody>
</table>

Table 2: Nondimensional parameter values.

<table>
<thead>
<tr>
<th>Case</th>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>in vacuo</td>
<td>Λ²</td>
<td>1</td>
<td>Solid-mechanics timescale ratio</td>
</tr>
<tr>
<td>FSI</td>
<td>Re</td>
<td>25.2</td>
<td>Reynolds number</td>
</tr>
<tr>
<td></td>
<td>St</td>
<td>1</td>
<td>Strouhal number</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>1.05 × 10⁻⁶</td>
<td>FSI parameter</td>
</tr>
<tr>
<td></td>
<td>ρ⁺_s/ρ⁺_f</td>
<td>84.2</td>
<td>Solid-to-fluid density ratio</td>
</tr>
<tr>
<td></td>
<td>Λ²</td>
<td>1.98 × 10⁻³ (1-d)</td>
<td>Solid-mechanics timescale ratio (1-d)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.23 × 10⁻³ (2-d)</td>
<td>Solid-mechanics timescale ratio (2-d)</td>
</tr>
</tbody>
</table>

3.1.4 Numerical procedure

To appreciate the physical scales of the problem we specify the input parameters in dimensional form and then derive their associated nondimensional quantities. To facilitate comparison with the more familiar linear Euler-Bernoulli formulation of the structural mechanics we choose the same beam parameter values as the original ‘Plate 2’ simulations in [11]. Additionally, both for our 1-d and 2-d beams, we require a thickness (h), which we choose such that h⁺/l⁺_{flexible} = 1/32, and a Poisson’s ratio (ν), which we obtain from [2]. This set of dimensional parameter values are summarized in Table 1 and form the reference for subsequent parametric variations. The corresponding values of the important nondimensional parameters required to set up the finite element problems (in vacuo and FSI) in oomph-lib are summarized in Table 2.

The problem is formulated using the open-source finite-element library oomph-lib [4, 5]. We use two-node Hermite beam elements for the 1-d beam and nine-node quadrilateral displacement-based solid mechanics (PVD) elements for the 2-d equivalent. Timestepping is performed using a Newmark scheme. The external traction used to produce the second eigenmode displacement of the beam is applied incrementally over a series of steady-state solutions. The external traction is then removed and the beam is free to oscillate. We have tested our code at various mesh densities and timestep sizes, and employed oomph-lib’s adaptive mesh refinement capabilities; e.g., see Figure 5(b).

3.2 Results and discussion

In the results that follow all parameter values are as per Tables 1 and 2 unless stated otherwise. Here we verify the structural codes against linear Euler-Bernoulli theory, beginning with the 1-d beam.

The bold curve in Figure 2 shows the second mode displacement profile corresponding to the applied load of T_{ext}, as predicted by the linear theory; the nondimensional y coordinate is normalized by the amplitude of the displacement η₀ (= η₀/l⁺_{flexible}). For a finite length 1-d beam one would expect a departure from this profile for increasing amplitudes as the curvature becomes more prominent. This is precisely what is observed in the four remaining curves corresponding to η₀ = 0.04, 0.1, 0.2 and
Figure 2: Approximations of the second in vacuo eigenmode at different amplitudes $\eta_0$ for the 1-d plate.

Figure 3: (a,b) Demonstration of the influence of amplitude on mode shape and (c) tip position for the 1-d plate model in vacuo.

0.4. For smaller $\eta_0$, the displacement predicted by the numerical code and the linear theory become indistinguishable, verifying the 1-d code for steady static solutions.

Figure 3 demonstrates what happens when the previous applied load is removed and the beam allowed to oscillate freely for the (a) $\eta_0 = 0.04$ and (b,c) $\eta_0 = 0.2$ cases. Consider Figure 3(a). The beam begins from the profile indicated by the bold green curve ($t^* = 0$, corresponding to the dotted curve in Figure 2) and the ensuing oscillations have the mode shape described by the series of black curves; the final snapshot at $t^* = 1.2$ sec is given by the bold red curve. Since the amplitude is sufficiently small the mode closely matches the well-known shape predicted by linear theory [e.g., see [2], Figure 2(b)]. Figure 3(b) shows the analogous result for larger amplitude oscillations that, as expected, deviate significantly from the linear mode. A time trace of the vertical position of the beam tip from Figure 3(b) is depicted in Figure 3(c). To verify the 1-d code for unsteady solutions we examine the eigenmode frequency. Figure 4 shows the ratio of the numerical results to the theoretical prediction over a range of amplitudes for the second and fourth modes. As the amplitude approaches the linear regime there is a convergence to the linear theory ($\omega/\omega_n \rightarrow 1$).

Adding a second dimension to the beam allows for stress variation across the thickness. Figure 5(a)
Figure 4: Verification of the dynamic response of the 1-d beam in vacuo—convergence of mode frequency $\omega$ to the linear theory at small amplitudes.

Figure 5: (a) Stress field in the 2-d beam in vacuo ranging from compressive (blue) tensile (red); (b) demonstration of mesh adapting around stress concentration at the clamped end.

demonstrates the compressive (blue) and tensile (red) stresses of the second eigenmode. In order to resolve the large stress gradients at the clamped end the mesh has automatically refined itself using the quad-tree procedure, as shown in Figure 5(b). Analysis of the eigenmode static profiles and oscillation frequencies reveals the same behaviour demonstrated for the 1-d beam in the limit of small amplitudes (Figures 2–4) provided that the beam thickness-to-length ratio is also small, with increasing deviation from this behaviour at larger ratios. The 2-d beam code has also been verified previously using the case of a uniform applied transverse load by comparing with the theoretical St Venant’s solution for the stress field (see http://oomph-lib.maths.man.ac.uk/doc/solid/airy_cantilever/html/index.html).

4 Beam in viscous channel flow: fluid-structure interaction

4.1 Method

4.1.1 Governing equations

When the beam is immersed in the viscous channel flow we choose the height of the channel as the characteristic length, $L^* = H^*$, the mean inlet velocity as the characteristic velocity, $U^* = U^*$, the natural timescale of the fluid flow as the characteristic time, $T^* = T^*_{fl} = H^* / U^*$, and use the viscous scale to define a characteristic pressure, $P^* = \mu^* U^* / H^*$. The fluid flow is then governed by the nondimensional Navier-Stokes equations

$$Re \left( St \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right],$$

(9)
and nondimensional continuity equation
\[
\partial u_i / \partial x_i = 0,
\]
(10)
where \( \text{Re} = \rho^* U^* H^*/\mu^* \) and \( \text{St} = (H^*/U^*)/\mathcal{T}^* = 1 \) are the Reynolds number and Strouhal number, respectively.

For the 1-d beam immersed in fluid flow the PVD is as per (1) but the traction vector is the summed pressure and viscous loads of the fluid on its ‘top’ and ‘bottom’ faces,
\[
T_i = Q \left\{ \left[ p_{\text{top}} \hat{N}_i^{[\text{top}]} - \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]_{\text{top}} \hat{N}_j^{[\text{top}]} \right\} + \\
\left\{ \left[ p_{\text{bottom}} \hat{N}_i^{[\text{bottom}]} - \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]_{\text{bottom}} \hat{N}_j^{[\text{bottom}]} \right\},
\]
(11)
for \( i = 1, 2 \), where \( \hat{N}_i^{[\text{top}]} \) and \( \hat{N}_i^{[\text{bottom}]} \) are the outer unit normals on the top and bottom faces of the deformed beam. The nondimensional parameter
\[
Q = \frac{\mu^* U^*}{E^* H^*}
\]
(12)
is the ratio of the fluid pressure scale, \( \mu^* U^*/H^* = \bar{P}^* \), used to nondimensionalise the Navier-Stokes equations, to the beam’s effective elastic modulus, \( E^* \) \((= S^*)\), used to nondimensionalise the PVD equation. Therefore \( Q \) indicates the strength of the fluid-structure interaction. In particular, if \( Q = 0 \) the beam deformation is not affected by the fluid flow.

For the fluid-immersed 2-d beam the traction vector is
\[
T_i = Q \left\{ p \hat{N}_i - \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \hat{N}_j \right\}
\]
(13)
where
\[
Q = \frac{\mu^* U^*}{E^* H^*}.
\]
(14)

Note that when the beam is immersed in channel flow the solid mechanics timescale ratio can be expressed as \( \Lambda^2 = (\rho^*/\rho^*) \text{Re} \text{ St}^2 Q \).

### 4.1.2 Boundary and initial conditions

The setup for the beam is as per §3.1.3 but additional constraints are required for the fluid. No slip conditions are applied on all walls, which for the flexible surfaces of the beam are
\[
u = \text{St} \frac{\partial R(\xi, t)}{\partial t}.
\]
(15)
for the 1-d beam and
\[
u = \text{St} \frac{\partial R(\xi_{\text{top, tip, bottom}}, t)}{\partial t}
\]
(16)
for the ‘top’, ‘tip’ and ‘bottom’ surfaces of the 2-d beam (parametrised by local Lagrangian coordinates). The two inlets have a parabolic velocity profile and the outflow is parallel. The initial condition for the fluid is zero flow.

### 4.1.3 Numerical procedure

The reference set of input parameter values are summarised in Tables 1 and 2. The additional parameters for the fluid are again taken from ‘Plate 2’ simulations in [11], except here the mean inlet velocity is an order of magnitude smaller.
Further to the details provided in §3.1.4, the fluid equations are discretised using nine-node quadrilateral Taylor-Hood elements and timestepping is performed using a BDF scheme. The FSI problem is discretised monolithically and the Newton-Raphson method is used to solve the nonlinear system of equations (specified by the global Jacobian matrix and the global residual vectors), employing the SuperLU direct linear solver within the Newton iteration. The flow is ramped up from zero over a period of 20 time steps.

4.2 Results and discussion

Having established the credibility of the structural models we now couple these to the fluid model and investigate their interaction. Once again beginning with the 1-d beam, consider Figure 6. Figure 6(a) shows the initial mesh with the central rigid wall drawn in red and the beam in blue. This structured mesh deforms according to the movements of the beam. Plotted in Figure 6(b) are a series of axial velocity ($u_1$) fields at consecutive time steps showing the beam undergoing an oscillation, and Figure 6(c) shows the corresponding pressure field at the last time step. This simulation begins in a mode 2 configuration with a displacement amplitude of $\eta_0 = 0.01$, which grows exponentially in time through a series of oscillations, reaching some twenty times that amplitude by the cycle depicted in Figure 6.

The analogous plots for the 2-d beam FSI model are shown in Figure 6(d)–(f), although the magnitude rather than axial component of velocity is plotted. The beam thickness ratio is again 1/32 but this now relates to a second geometrical dimension rather than simply as a means of approximating the contribution of flexural rigidity. For brevity we now focus on this rather than the 1-d beam FSI model noting that we
observe similar behaviour in both. Although Figure 6(d)–(f) clearly depicts a large-amplitude oscillation one can better appreciate the instability of the system from Figure 7(a), where the vertical displacement of the tip of the beam centreline has been plotted as it varies in time (bold line); the second in vacuo eigenmode from linear theory is also plotted (thin line) to show how the FSI augments the amplitude and frequency of the structural oscillations. A flutter instability is present as evident by the amplitude growth of the oscillations. The reduction in oscillation frequency by 4% compared to the in vacuo linear theory is largely accounted for by the fluid traction. By reducing the inlet velocity by an order of magnitude the same initial conditions produce oscillations that decay very rapidly, as demonstrated in Figure 7(b). This shows the existence of a critical flow speed (or Re) for flutter onset.

The stability trend with inlet velocity magnitude can be gleaned from Figure 8 in which the centreline tip amplitude is plotted in logarithmic scale against time. The curve with ‘×’ markers denotes a simulation for which the FSI is switched off by setting $Q = 0$; i.e., the beam does not ‘feel’ the fluid traction and so is able to oscillate freely, effectively serving as a mechanism for deforming a time-dependent fluid domain. The amplitude remains constant at the initial value, as would be expected. Using this as a neutral stability reference we can interpret the remaining cases for which $Q$ assumes its natural value.

As the inlet velocity is reduced from $10U$ down to $U/10$ the amplitude growth rate reduces, eventually becoming less than unity when stability is achieved. Linearly interpolating the growth rates predicts a critical Re of around 4 ($U^* = 0.015$ m/s) for this channel geometry. The curves denoted by ‘2U’ (doubled inlet velocity) and ‘$U, 2H$’ (doubled channel width) have the same Re but the former is unstable while the latter is stable. This is because increasing the channel width reduces the Bernoulli effect between the beam and the channel walls.

Space limitations prevent us from presenting a detailed stability analysis but to demonstrate the capability to do so, we include results for a beam of double thickness but the same mass per unit area (‘$U, 2h$’ in Figure 8), which suggests beam thickness may be a stabilising factor, and a beam of 10 times the density (‘$U, 10\rho_s$’, same figure), indicating that a uniform increase in inertia may be destabilising.
5 Conclusion

We have presented a model of a finite-thickness cantilevered flexible plate interacting with a viscous channel flow. A geometrically nonlinear formulation of the solid mechanics was employed. This model was developed using oomph-lib, an open-source finite-element library. We have demonstrated both stable and unstable motions of the flexible plate with the latter involving large-amplitude flutter-type oscillations. This work provides the foundation for a more anatomically accurate analysis of the mechanisms underlying obstructive breathing disorders.

6 Acknowledgement

We acknowledge the support of both the Australian Research Council through the project DP0559408 and the WA State Centre of Excellence in eMedicine in the continuing project ‘Airway Tomography Instrumentation’.

References